GEC223: FLUID MECHANICS

MODULE 5: BOUNDARY LAYERS IN PIPE AND EFFECTS OF SURFACE ROUGHNESS

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When real (viscous) fluid flows past a stationary solid boundary, some liquids adhere to the solid boundary, which is a condition called no-slip condition. The next layer undergoes retardation and further retards the subsequent adjacent layers of the fluid. The velocity of the fluid in adjoining layers increases rapidly from zero at the boundary surface and approaches velocity of mainstream at the centre. The layer adjacent to the boundary is known as boundary layer.

Boundary layer is formed whenever there is relative motion between a fluid and surrounding boundary.

Since the shear stress, $\tau_0 = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$ from Newton's law of viscosity, the fluid exerts a shear stress on the boundary and the boundary exerts an equal but opposite force on the fluid known as shear resistance.

Two regions are noticed:

1. A thin layer adjoining the boundary, called a boundary layer where viscous shear takes place.
2. A region outside the boundary layer where flow behaviour is like that of ideal fluid and the potential flow theory is applicable.

CHARACTERISTICS OF BOUNDARY LAYER

Consider a parallel flat plate parallel to the fluid of velocity U.

1. Thickness of boundary layer layer, δ, increases as distance from the leading edge x increases.
2. δ decreases as fluid velocity, U, increases.
3. τ_0 decreases as x increases . Recall, $\tau_0 = \cong \mu \left(\frac{U}{\delta}\right)$
4. Critical value of $\frac{Ux}{\nu}$ governs the transition from laminar boundary condition to turbulent boundary condition. This critical value depends on:
   i. Surface roughness
   ii. Pressure gradient
   iii. Plate curvature

iv. Temperature difference between fluid and boundary

v. Turbulence in ambient flow

5. If $\frac{Ux}{v}$ < 5 x 10^5, boundary layer is laminar. For laminar layer, velocity distribution is parabolic.

   If $\frac{Ux}{v}$ > 5 x 10^5, boundary layer is turbulent. For turbulent layer, velocity distribution follows logarithmic or follows power law.
6. δ increases as kinematic viscosity, ν, increases.
7. When U increases in the downward direction, the boundary layer growth reduces.
8. When U decreases in the downward direction, the boundary layer growth is faster and is susceptible to separation.

BOUNDARY LAYER THICKNESS, δ

The nominal boundary layer thickness is measured in three ways:

1. Displacement thickness, (δ^*)
2. Momentum thickness (θ)
3. Energy thickness(δ_e)

1. Displacement thickness is the distance, measured perpendicular to the boundary, by which the main free stream is displaced on account of formation of boundary layer.

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

2. Momentum thickness is the distance through which the total loss of momentum per second be equal to if it were passing a stationary plate.

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

3. Energy thickness is the distance, measured perpendicular to the boundary of the solid, by which the boundary should be displaced to compensate for the reduction in kinetic energy, KE, of the flowing fluid on account of boundary layer formation.

$$\delta_e = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

Consider fluid flow through a pipe. The shear stress distribution and velocity distribution curve is as shown below.

From Newton's law of viscosity,

$$\tau = \mu . \frac{du}{dy} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} 1$$

The radial distance, r, is related to y be relation:

y = R – r or dy = - dr

Equation 1 becomes $\tau = \mu.\frac{du}{-dr} = -\mu.\frac{du}{dr}$

$$-\mu.\frac{du}{dr} = -\frac{\partial P}{\partial x}.\frac{r}{2}$$

$$du = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) r.dr \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 2$$

Integrating w.r.t. "r"we get

$$u = \frac{1}{4\mu}.\frac{\partial P}{\partial x}r^2 + C \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 3$$

where C is the constant of integration obtained as follows:

At r = R, u = 0 (i.e. at the pipe wall)

$$\therefore 0 = \frac{1}{4\mu}.\frac{\partial P}{\partial x}r^2 + C$$

where $C = -\frac{1}{4\mu}.\frac{\partial P}{\partial x}R^2$ ________________________ 4

Substituting for C in equation 3 gives

$$u = \frac{1}{4\mu}.\frac{\partial P}{\partial x}r^2 - \frac{1}{4\mu}.\frac{\partial P}{\partial x}R^2$$

$$\mathbf{u = -\frac{1}{4\mu}.\frac{\partial P}{\partial x}(R^2 - r^2)} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{Equation 5}$$

This shows that the velocity distribution curve is a parabola.

The maximum velocity occurs at the centre where r = 0

$$\therefore U_{max} = \mathbf{-\frac{1}{4\mu}.\frac{\partial P}{\partial x}.R^2} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{Equation 6}$$

Combining equations 5 and 6,

$$\frac{u}{U_{max}} = \frac{-\frac{1}{4\mu}.\frac{\partial P}{\partial x}(R^2 - r^2)}{-\frac{1}{4\mu}.\frac{\partial P}{\partial x}.R^2}$$

$$\frac{u}{U_{max}} = \frac{R^2 - r^2}{R^2} = 1 - \left(\frac{r}{R}\right)^2$$

$$\therefore \boldsymbol{u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{Equation 7}$$

This equation applies for velocity distribution for laminar flow through pipes.

To calculate the discharge:

Discharge through an elementary ring of thickness dr at a radial distance r is given by:

$$dQ = u \times 2\pi r \times dr = \boldsymbol{u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]}.2\pi r.dr$$

∴ Total discharge through the entire pipe,

$$Q = \int dQ = \int_0^R u_{max}\left[1-\left(\frac{r}{R}\right)^2\right].2\pi r.dr$$

$$= 2\pi u_{max}\int_0^R\left(r-\frac{r^3}{R^2}\right)dr = 2\pi u_{max}\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R$$

$$= 2\pi u_{max}\left[\frac{R^2}{2}-\frac{R^4}{4R^2}\right] = 2\pi u_{max}\left[\frac{R^2}{2}-\frac{R^2}{4}\right]$$

$$\mathbf{Q = 2\pi u_{max}\left[\frac{R^2}{4}\right] = \frac{\pi}{2}u_{max}R^2}$$ ________________________Equation 8

Average velocity of flow, $\bar{u} = \frac{u_{max}}{2}$ ____________________Equation 9

This implies, $u_{max} = 2\bar{u}$

This shows that the average velocity is half of the maximum velocity.

Substituting the value of u_{max} from equation 6 in equation 9 gives,

$$\bar{u} = -\frac{1}{4\mu}.\frac{\partial P}{\partial x}.R^2.\frac{1}{2} = -\frac{1}{8\mu}.\frac{\partial P}{\partial x}.R^2$$

$$-\partial P = \frac{8\mu\bar{u}}{R^2}.\partial x$$

The pressure difference between two sections 1 and 2 at distances x_1 *and* x_2 in a pipeline as shown below:

$$-\int_{P_1}^{P_2}\partial P = \frac{8\mu\bar{u}}{R^2}\int_{x_1}^{x_2}\partial x$$

$$(P_1 - P_2) = \frac{8\mu\bar{u}}{R^2}(x_2 - x_1) = \frac{8\mu\bar{u}L}{R^2}$$

$$\therefore P_1 - P_2 = \Delta P = \frac{8\mu\bar{u}L}{R^2}$$ _________________Equation 10

Equation 10 is known as Hagen-Poiseuille equation.

Since $R = \frac{D}{2}$ substitution in equation 10 gives,

$$P_1 - P_2 = \frac{8\mu\bar{u}L}{\frac{D^2}{4}}$$

This implies $P_1 - P_2 = \frac{32\mu\bar{u}L}{D^2}$ ________________________Equation 11

Example 1

An oil of viscosity 9 poise and specific gravity 0.9 is flowing through a horizontal pipe of 60mm diameter. If the pressure drop in 100m length of pipe is 1800KN/m^2, determine:

i. The rate of flow of oil
ii. The centre-line velocity
iii. The total frictional drag over 100m length
iv. The power required to maintain the flow
v. The velocity gradient at the pipe wall
vi. The velocity and shear stress at 8mm from the wall

Solution:

Viscosity of the oil, μ = 9 poise = $\frac{1}{10}$ x 9 = 0.9NS/m^2

Specific gravity of oil = 0.9

Diameter of pipe, D = 60mm = 0.06m

∴ Area of pipe, A = $\frac{\pi D^2}{4}$ = 0.002827m^2

∴ Pressure drop in 100m length of pipe, ΔP = 1800KN/m^2

i. Rate of flow, Q

Recall, $P_1 - P_2 = \Delta P = \frac{32\mu \bar{u} L}{D^2}$ where $\bar{u}$ = average velocity

1800 x 10^3= $\frac{32 \ x \ 0.9 \ x \bar{u} \ x \ 100}{0.06^2}$

$\bar{u} = \frac{1800 \ x \ 10^3 \ x \ 0.06^2}{32 \ x \ 0.9 \ x \ 100}$ = 2.25m/s

Reynolds number, Re = $\frac{\rho V D}{\mu} = \frac{0.9 \ x \ 1000 \ x \ 2.25 \ x \ 0.06}{0.9}$ = 135 < 2000

∴ Flow is laminar.

∴ Rate of flow, Q = A. $\bar{u}$ = 0.002827 x 2.25 = 0.00636 m^3/s = **6.36 lit/s**

ii. Centre-line velocity, u_{max} = 2$\bar{u}$ = 2 x 2.25 = **4.5m/s**

iii. Total friction drag over 100m length, F_D

Recall, $\tau_0 = \frac{\partial P}{\partial x} . \frac{R}{2}$ and

$\frac{-\partial P}{\partial x}$ = - $\frac{P_2 - P_1}{x_2 - x_1}$ = $\frac{P_1 - P_2}{L}$ = $\frac{\Delta P}{L}$ = $\frac{1800 \ x \ 10^3}{100}$

∴ - $\frac{\partial P}{\partial x}$ = 18000

∴ τ_0 = 18000 x $\frac{0.06/2}{2}$ = 270N/m^2

∴ F_D = τ_0 x πDL = 270 x π x 0.06 x 100 = 5089N = **5.089KN**

iv. Power required to maintain the flow, P = F_D x $\bar{u}$ = 5.089 x 2.25 = **11.45KW**

Alternatively, P can be obtained as P = Q . ΔP = 0.00636 x1800 = 11.45KW

v. Velocity gradient at the pipe wall, $\left(\frac{du}{dy}\right)_{y=0}$

Recall, τ_0 = μ. $\left(\frac{du}{dy}\right)_{y=0}$

$\left(\frac{du}{dy}\right)_{y=0}$ = $\frac{\tau_0}{\mu}$ = $\frac{270}{0.9}$ = **300** s^{-1}

vi. Velocity and shear stress at 8mm from the wall

Recall, u = $-\frac{1}{4\mu} \cdot \frac{\partial P}{\partial x}\ (R^2 - r^2)$

y = 8mm = 0.008m

y = R – r

0.008 = 0.03 – r

This implies r = 0.03 – 0.008 = 0.022m

$u_{8mm} = -\frac{1}{4 \ x \ 0.9} \cdot \frac{1800 \ x \ 10^3}{100}\ (0.03^2 - 0.022^2)$

u_{8mm} = 2.08m/s

Also, since $\frac{\tau}{r} = \frac{\tau_0}{R}$

This implies, τ_{8mm} = r x $\frac{\tau_0}{R}$ = 0.022 x $\frac{270}{0.03}$

τ_{8mm} **= 198KN/m^2**

Example 2

A pipe 60 mm diameter and 450 m long slopes upwards at 1 in 50. An oil of viscosity 0.9 Ns/m^2 and specific gravity 0.9 is required to be pumped at the rate of 5 litres/sec.

i. Is the flow laminar?
ii. What pressure difference is required to attain this condition?
iii. What is the power of the pump required assuming an overall efficiency of 65%?
iv. What is the centre-line velocity and velocity gradient at the pipe wall?

Solution

Diameter of pipe, D= 60mm = 0.06m

Area of pipe, A = $\frac{\pi D^2}{4}$ = $\frac{\pi \, x 0.06^2}{4}$ = 0.00283 m^2

Length of pipe, L = 450m

Slope = 1 in 50

Viscosity of oil, μ = 0.9Ns/m^2

Weight density, ω = ρg = 0.9 x 9810 = 8829N/m^2

Discharge, Q = 5 litres/s = 0.005m^3/s

Note: 1 m^3 = 1000 litres

Overall efficiency, η_0 = 65%

i. Is the flow laminar?

Average velocity, $\bar{u}$ = $\frac{Q}{A}$ = $\frac{0.005}{0.00283}$ = 1.767 m/s

Reynolds number, Re = $\frac{\rho VD}{\mu}$ = $\frac{(0.9\, x\, 1000)\, x\, 1.767\, x\, 0.06}{0.9}$

Re = 106 < 2000

∴ **Flow is laminar**

ii. Pressure difference required

Applying Bernoulli's theorem between sections 1 and 2 gives,

$$\frac{P_1}{\omega} + \frac{V_1{}^2}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2{}^2}{2g} + h_f$$

$$\frac{P_1}{\omega} + \frac{V_1{}^2}{2g} + 0 = \frac{P_2}{\omega} + \frac{V_2{}^2}{2g} + \frac{1}{50}\, x\, 450 + \frac{32\mu\bar{u}L}{D^2\, x\, \omega}$$

$$\frac{P_1 - P_2}{\omega} = 9 + \frac{32\mu\bar{u}L}{D^2\, x\, \omega} \quad \text{(Since } V_1 = V_2 \text{ and } Z_1 = 0\text{)}$$

$$P_1 - P_2 = 9\omega + \frac{32\mu\bar{u}L}{D^2} = 9 \text{ x } 8829 + \frac{32\, x\, 0.9\, x 1.767\, x\, 450}{0.06^2}$$

= 79461 + 6.36 x 10^6 = **6.44 x 10^6N/m^2**

∴ Pressure difference, $P_1 - P_2 = 6.44\ MN/m^2$

iii. Power of the pump, $P_{required}$ = Q $(P_1 - P_2)$ = 0.005 x 6.44 x 10^3 = 32.2KW

Power of the pump taking into cognisance the overall efficiency, P = $\frac{P_{required}}{\tau_0}$ = $\frac{32.2}{0.65}$ = **49.54 KW**

iv. Centre-line velocity, μ_{max} = $2\bar{u}$ = 2 x 1.767 = 3.534 m/s

Velocity gradient at the pipe wall, τ_0 = - $\frac{\partial P}{\partial x}$. $\frac{R}{2}$ = $\frac{6.44 \, x \, 10^6}{450}$. $\frac{R}{2}$ = 214.67 N/m^2

Recall, $\tau_0 = \mu . \left(\frac{du}{dy}\right)_{y=0}$

$\therefore \left(\frac{du}{dy}\right)_{y=0} = \frac{\tau_0}{\mu} = \frac{214.67}{0.9}$ = **238.5** s^{-1}

NOTE: For circular pipes,

i. Kinetic Energy Correction factor, α = 2.0

where $\alpha = \frac{K.E/s \text{ based on actual velocity}}{K.E/s \text{ based on average velocity}}$

ii. Momentum correction factor, $\beta = \frac{Momentum/s \text{ based on actual velocity}}{Momentum/s \text{ based on average velocity}}$

iii. For viscous flow, the coefficient of friction, f, is given by:

$$f = \frac{16}{Re}$$

where Re = $\frac{\rho VD}{\mu} = \frac{VD}{v}$

where μ = dynamic viscosity and v = kinematic viscosity of fluid